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Parsimony and Causality

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Abstract This paper takes issue with the current tendency in the literature on Qualitative Comparative Analysis (QCA) to settle for so-called *intermediate* solution formulas, in which parsimony is not maximized. I show that there is a tight conceptual connection between parsimony and causality: only maximally parsimonious solution formulas reflect causal structures. However, in order to maximize parsimony, QCA—due to its reliance on Quine-McCluskey optimization (Q-M)—is often forced to introduce untenable simplifying assumptions. The paper ends by demonstrating that there is an alternative Boolean method for causal data analysis, *viz.* Coincidence Analysis (CNA), that replaces Q-M by a different optimization algorithm and, thereby, succeeds in consistently maximizing parsimony without reliance on untenable assumptions.

1 Introduction

Sufficient and necessary conditions—the primary search targets of Boolean (or set-theoretic or configurational) methods of causal inference—tend to feature redundant elements, i.e. factors such that, if they are eliminated, the remaining conditions are still sufficient and/or necessary for corresponding outcomes. For the methodological framework of Qualitative Comparative Analysis (QCA), Ragin and Sonnett (2005) distinguish three different search strategies researchers may adopt when eliminating redundant elements from Boolean solution formulas (Boolean models[1]). In case of limitedly diverse data, only the most liberal strategy is able to eliminate all redundancies. Yet, as this strategy typically requires a host of counterfactual simplifying assumptions, which are often difficult or impossible to justify, it has recently become common practice in QCA studies to settle for so-called *intermediate* solution formulas with some redundancies remaining (cf. Ragin and Rihoux 2004; Ragin 2008b, ch. 9; Schneider and Wagemann 2012, chs. 6, 8; Skaaning 2011).[2] This is usually

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[1] In this paper, the notion of a *model* is used in terms of a full specification of the functional dependencies among endogenous and exogenous factors. In the QCA literature, a model is sometimes (e.g. Wright and Boudet 2012) simply taken to be a selection of endogenous and exogenous factors (without a specification of functional dependencies).

[2] Some studies even settle for so-called *conservative* solution formulas with all redundancies remaining (cf. Grofman and Schneider 2009).

legitimized with recourse to principles as Occam's Razor, according to which parsimony is a mere pragmatic virtue of causal models to the effect that, if easily available, parsimonious models are to be preferred over more complex ones, but if parsimony comes at a high price it can be dispensed with.

This paper shows that intermediate (and conservative) solution formulas cannot be causally interpreted. Subject to e.g. Mackie's (1974) INUS-theory of causation, to which representatives of QCA explicitly subscribe (cf. e.g. Mahoney and Goertz 2006; Schneider and Wagemann 2012), causes are Boolean difference-makers for their effects. Yet, elements of solution formulas that can be eliminated without relationships of sufficiency and necessity thereby being affected make no difference to corresponding outcomes and, thus, do not cause the latter. That is, parsimony of solution formulas is much more than a mere pragmatic virtue of causal models; rather, there exists a tight conceptual connection between parsimony and causality. Only maximally parsimonious solution formulas can represent causal structures. Accordingly, researchers that want to apply QCA for the purpose of causal data analysis or of testing causal hypotheses must not content themselves with intermediate solution formulas.[3]

The structure of the paper is as follows. In section 2, I review the details of the theory that accounts for causation in terms of Boolean difference-making and that underlies all Boolean causal reasoning. On the basis of an inverse search that puts the available search strategies of QCA to the test, section 3 then shows that only the most liberal strategy—which maximizes parsimony, yet tends to call for unjustifiable simplifying assumptions—succeeds in uncovering causal dependencies as defined by that theory. It follows that users of QCA face a dilemma: either they introduce untenable simplifying assumptions and, thereby, maximize parsimony and secure the causal interpretability of resulting solution formulas, or they base their inferences on a sound and tenable assumptive basis and, thereby, limit the applicability of QCA to non-causal purposes. In section 4, it will turn out that the culprit for this dilemma is QCA's reliance on Quine-McCluskey optimization (Q-M) to eliminate redundancies from solution formulas. The paper ends by arguing that by replacing Q-M by a different optimization algorithm—one that is custom-built for the discovery of causal structures—so-called *Coincidence Analysis* (CNA) (cf. Baumgartner 2009a; 2009b) succeeds in consistently maximizing parsimony without reliance on untenable assumptions.

2 Causes as Boolean Difference-Makers

Before we can investigate which search strategies of QCA yield solution formulas that can be taken to reproduce causal structures, we have to clarify what *causation* or *causal dependence* means in the context of Boolean data analysis. Very roughly put, the (philosophical) literature concerned with analyzing causal notions provides two different types or traditions of theories of causation: the first type is constituted by so-called *difference-making* theories and the second by *transference* and *power* theories. Prototypical examples of the first type are Suppes (1970), Lewis (1973), Mackie (1974), or more recently Woodward (2003);

[3] There also exist non-causal applications of QCA. For instance, Mendel and Korjani (2012) use QCA as a method for *linguistic summarization*, which is a data mining approach to extract patterns from databases. The goal of this approach is not to generate causal knowledge, causal explanations, or to test causal hypotheses; rather, it seeks to better understand and communicate about data. For such a purpose, solution formulas that are not maximally parsimonious may be very useful. Moreover, intermediate solutions may be informative with respect to correlations and associations among investigated factors; cf. e.g. Grant, Morales, and Sallaz (2009, 344-345) who very explicitly abstain from causally interpreting intermediate solutions (though for different reasons than the ones advanced in this paper).

well-known examples of the second type are Dowe (2000) or Mumford and Anjum (2011).[4] In a nutshell, difference-making theories stipulate—as their name suggests—that causes are characterized by their property of making some sort of difference to their effects, where the relevant sort of difference-making is variably specified in different theories. Power theories, by contrast, take the characteristic feature of causal dependencies to consist in some sort of physical relation connecting causes to their effects, for instance, the transference of energy or momentum from the cause to the effect or the cause's exertion of power over the effect.

It is clear that Boolean methods of data analysis do not scrutinize the physical relation between causes and effects. Consequently, they do not search for causal dependencies as defined by transference or power theories. Rather, Boolean methods search for causal dependencies as defined by difference-making theories. More specifically, as their primary search targets are sufficient and necessary conditions, Boolean methods must be seen to presuppose a notion of causation according to which causes are difference-makers within sufficient and necessary conditions of their effects. The most well-known theory in that tradition is Mackie's (1974) so-called *INUS-theory* of causation. Subject to that account, a cause is an INUS-condition of its effect, *viz.* an *i*nsufficient but *n*on-redundant part of an *u*nnecessary but *s*ufficient condition of the latter (Mackie 1974, 62). That is, a cause or, in Mackie's terminology, a *causally relevant factor* A for an effect E (typically) is neither itself sufficient nor necessary for E; rather, relative to some fixed configuration of background conditions, A is a non-redundant element of a configuration of factors, say, ABC which, in turn, is sufficient but not necessary for E. That is, there may be alternative configurations, say, DGH and JKL that can likewise cause E—even in the absence of ABC. Relative to that theoretical background, A being a difference-maker for E amounts to this: there exist circumstances ϕ, *viz.* when BC is given and neither DGH nor JKL are given, such that A makes a difference to the occurrence of E in ϕ, i.e. E occurs in ϕ if, and only if, A occurs in ϕ. In other words, in circumstances of type ϕ, A is a means to control or manipulate E: by bringing about (suppressing) A, E is brought about (suppressed).

In order to see that only the most parsimonious solution formulas of Boolean methods succeed in capturing causation as defined along these lines, we have to delve more deeply into the conceptual details of a theory that spells out causation in terms of Boolean difference-making. As a brief reminder, let me begin with the relevant formal background from Boolean algebra (for more details cf. Thiem et al. ms). The core Boolean operation that allows for representing (deterministic) causal dependencies is the *implication* or *subset* operator, which in the context of Boolean methodologies is alternatively expressed in a logical ('$\rightarrow$') or set-theoretical ('$\subset$') formalism (cf. Schneider and Wagemann 2012, part I). For mere conventional reasons I shall in the following give preference to the logical mode of expression. On the basis of the implication operator the two core notions of Boolean causal modeling can then be defined: the notions of sufficiency and necessity. A is said to be *sufficient* for E if, and only if, it holds that if A is the case, then E is the case as well, or formally $A \rightarrow E$. By contrast, A is *necessary* for E if, and only if, it holds that if E is the case, then A is the case as well, or formally $E \rightarrow A$. Furthermore, that A is both sufficient and necessary for E is expressed with the biconditional operator '$\leftrightarrow$': $A \leftrightarrow E$.

As causal structures in the world we live in hardly ever involve only one cause and one effect, three further Boolean operations are needed to capture the whole complexity of ordinary causal structures: conjunction, disjunction and negation. A causally relevant factor A typically only determines its effect E if other factors as B *and* C are given as well. I shall

[4] For a recent illustration of the debates between these two theoretical camps see Glynn (2013) and Ney (2009).

subsequently express conjunction, *viz.* “and”, by mere concatenation of factors: $ABC \rightarrow E$. Moreover, an effect E can be brought about along alternative causal routes, for instance by the configuration ABC *or* by DGH *or* by JKL. Disjunction, *viz.* “or”, is expressed by ‘$\vee$’: $ABC \vee DGH \vee JKL \rightarrow E$. Finally, not only the presence but also the absence of a factor may be causally relevant or may be caused, that is, causally relevant factors and effects must be negatable. The negation of A, i.e. ‘not A’, will be written $\overline{A}$. Overall, thus, an exemplary Boolean representation of a more realistically complex causal structure would be this:

$$A\overline{B}C \vee DG\overline{H} \vee \overline{J}KL \leftrightarrow E \tag{1}$$

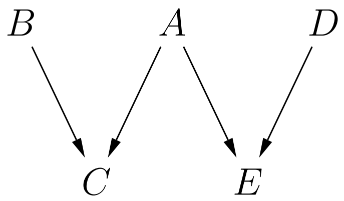
(1) represents a causal structure such that the effect E has exactly three alternative complex causes: $A\overline{B}C$, $DG\overline{H}$, $\overline{J}KL$. According to (1), each of these conjunctions is sufficient for E and their disjunction is jointly necessary for E, meaning that E does not occur if not at least one of its three alternative causes is given.[5]

However, only a very small subset of all dependencies of sufficiency and necessity actually reflect causal dependencies. The reason is that the relationships of sufficiency and necessity are *monotonic*, meaning that when A is sufficient for E, it follows (on mere logical grounds) that AX is also sufficient for E, and when A is necessary for E, it follows that $A \vee X$ is also necessary for E, where X in both cases stands for any arbitrary factor. As a consequence, most complex sufficient and necessary conditions involve a host of factors, whose removal from the conditions does not change their status as sufficient and necessary conditions. Factors that can be removed from conditions such that the latter's sufficiency and necessity remains unaltered are redundant and, thus, are not Boolean difference-makers and, therefore, are not causes.

In the following I illustrate the requirement of removing all redundant elements from sufficient and necessary conditions before the latter can be causally interpreted, first, with a concrete example from social science and, second, with an abstract common cause structure. Suppose that a high GNP (G) in combination with strong left parties (L) and a low ratio of foreign population ($\overline{F}$) are jointly sufficient for a high level of education (E), i.e. $GL\overline{F} \rightarrow E$. In words: whenever a country has a high GNP, strong left parties, and a low ratio of foreign population, it also has a high level of education. If that holds, it of course also holds that whenever a country has a high GNP, strong left parties, a low ratio of foreign population, *and a blond prime minister* (B), it also has a high level of education, or formally: $GL\overline{F}B \rightarrow E$. Hence, even though the prime minister's hair color makes no difference whatsoever to E and is thus redundant for E, B may be part of a sufficient condition of E. The fact that $GL\overline{F}B$ clearly is no complex cause of E shows that being part of a sufficient condition is not sufficient for being a cause. $GL\overline{F}B$ is not a cause of E because B can be removed without the remaining condition $GL\overline{F}$ losing its sufficiency for E.

Next, consider the common cause structure depicted in graph (a) of figure 1. In this structure, C and E are two parallel effects of the common cause A. In addition, there exists one further alternative cause for C and E each: B for C and D for E. For simplicity, all

[5] Usually, in the prose around solution formulas in QCA studies only necessary conditions that consist of single factors are explicitly labeled “necessary conditions”. (A commendable exception is Bol and Luppi (2013), who systematize the search for complex necessary conditions within the QCA framework.) In fact, however, every QCA solution formula that identifies complex sufficient conditions for the absence of an effect is tantamount to a solution formula that identifies a complex necessary condition for the presence of the effect (by contraposition), and vice versa. For instance, a formula that identifies AC and $\overline{D}$ as two alternative sufficient conditions for $\overline{E}$ is logically equivalent to a formula that identifies $\overline{A}D \vee \overline{C}D$ as necessary condition for E. Disjunctive necessary conditions can be interpreted as imposing restrictions on the space of alternative causes of an effect: in the structure represented by (1), E has exactly three alternative causes.



(a)

case	A	B	C	D	E
c_1	1	1	1	1	1
c_2	1	1	1	0	1
c_3	1	0	1	1	1
c_4	1	0	1	0	1
c_5	0	1	1	1	1
c_6	0	1	1	0	0
c_7	0	0	0	1	1
c_8	0	0	0	0	0

(b)

Fig. 1 Graph (a) depicts an ordinary common cause structure, and table (b) lists all the configurations of the involved factors that are compatible with the structure in (a), i.e. (b) is the (ideal and complete) truth-table for (a).

five factors in that structure are assumed to be binary. Table (b) then lists all the empirically possible configurations of these factors. That means there are 8 types of cases that can be found empirically, given that the behavior of the five factors is regulated by the structure in graph (a). For instance, all five factors can be given in combination (case c_1) or D can be absent when all the other factors are present (c_2), and so forth.

Let us now inquire what combinations of the factors in the set $\{A, B, C, D\}$ are sufficient and/or necessary for E. Without drawing on Boolean methods it can easily be seen that in table (b) it holds that whenever A is given (cases c_1 to c_4) and whenever D is given (c_1, c_3, c_5, c_7), E is given as well. Hence, A and D are both individually sufficient for E; and as these sufficient conditions are constituted by single factors, they do not involve redundancies either. At the same time, though, table (b) reveals that the combination $\overline{B}C$ is also sufficient for E: whenever $\overline{B}C$ is given (c_3, c_4), E is given as well. Or formulated in causal terms, in cases when C is given without B, A must be given to account for C, for no effect occurs without any of its causes. But since A determines E in structure (a), it follows that $\overline{B}C$ is sufficient for E as well. Moreover, no element of $\overline{B}C$ can be removed without the condition losing its sufficiency for E: C alone is not sufficient for E, because in c_6 C is given while E is not; and $\overline{B}$ alone is not sufficient for E, because in c_8 $\overline{B}$ is given while E is not. That is, $\overline{B}C$ does not contain any redundant elements.

A, D and $\overline{B}C$ are all of the redundancy-free sufficient conditions that E has in table (b). Furthermore, all cases featuring E also feature A or D or $\overline{B}C$; or differently, whenever E is given, so are A or D or $\overline{B}C$, which means that the disjunction $A \vee D \vee \overline{B}C$ is necessary for E. Overall, the following holds:

$$A \vee D \vee \overline{B}C \leftrightarrow E \quad (2)$$

Even though (2) features sufficient conditions of E that have no redundant elements, (2) is a Boolean expression that does not reflect a causal structure, for $\overline{B}C$ is not a complex cause of E in structure (a). The reason is that the necessary condition $A \vee D \vee \overline{B}C$ contains a redundant disjunct, *viz.* $\overline{B}C$. If $\overline{B}C$ is removed, the remaining disjunction $A \vee D$ is still necessary for E, for it holds that whenever E is given in table (b), so is A or D:

$$A \vee D \leftrightarrow E \quad (3)$$

By contrast, the necessary condition of E contained in (3), *viz.* $A \vee D$, is free of redundancies. A itself is not necessary for E because in cases c_5 and c_7 E is given while A is not; neither is D alone necessary for E because in cases c_2 and c_4 E is given and D is not. Indeed,

(3) is a Boolean expression that corresponds to a causal structure, for A and D are exactly the two alternative causes of E in structure (a). All of this shows that Boolean expressions only reflect causal structures if all redundancies are rigorously removed from both sufficient and necessary conditions. Or in other words, if a Boolean expression contains redundant elements, it does not reflect a causal structure.

To further specify the account of difference-making that is relevant for Boolean methods of causal inference, let us introduce the notions of a *minimally sufficient* and a *minimally necessary condition*. A conjunction as AB that is sufficient for an outcome E is *minimally* sufficient for E if, and only if, no proper part of AB is itself sufficient for E, where a proper part of a conjunction is that conjunction reduced by at least one conjunct. That is, AB is minimally sufficient for E if, and only if, AB is sufficient for E and neither A nor B are themselves sufficient for E. Analogously, a disjunction as $A \vee B$ that is necessary for an outcome E is *minimally* necessary for E if, and only if, no proper part of $A \vee B$ is itself necessary for E, where a proper part of a disjunction is that disjunction reduced by at least one disjunct. That is, $A \vee B$ is minimally necessary for E if, and only if, $A \vee B$ is necessary for E and neither A nor B are themselves necessary for E.

For factors that are contained in minimally sufficient and necessary conditions of an outcome there exist configurations in which these factors make a difference to the occurrence of that outcome. For example, compare the configurations c_2 and c_6 in table (b): in c_2 the factor A is present and in c_6 it is absent, whereas all other factors apart from the outcome E remain the same in both c_2 and c_6; in accordance with A, E is present in c_2 and absent in c_6. Hence, in c_2 and c_6 the presence and absence of A makes a difference to the presence and absence of E. For redundant elements of sufficient and necessary conditions, no such difference-making configurations exist. All of this yields the following account of Boolean difference-making:

Boolean difference-making (BD): A factor A is a Boolean difference-maker of an outcome E if, and only if, A is contained in a minimally sufficient condition AX of E such that AX, in turn, is contained in a minimally necessary condition of E.

Being a Boolean difference-maker is necessary but not sufficient for being a cause. Boolean difference-making is relative to analyzed data: for instance, C is not a Boolean difference-maker of E in table (b), but in a table that results from (b) by adding, say, the configuration $\overline{A}\overline{B}C\overline{D}E$ the factor C would be such a difference-maker. As is well-known, analyzed data may fail to reflect causal structures for various reasons: the data may be confounded by unmeasured causes, it may be miscalibrated, too fragmentary, or result from measurement errors etc. Relative to deficient data, factors may be Boolean difference-makers of an outcome E without being causes of E. Data deficiencies are typically exposed by expanding the set of analyzed factors in further studies of the same phenomenon, by conducting robustness tests, by recalibrations, remeasurements, or by mere replication studies etc. Only Boolean difference-makers in data that meet required quality standards can reliably be inferred to be causes (cf. Baumgartner 2008; 2013).

Of course, due to the fact that in contexts of causal data analysis researchers typically investigate causal structures which are not completely known, compliance with those quality standards often cannot be conclusively ascertained. Accordingly, inferring that A causes E normally involves an *inductive risk*, i.e. causal inferences are prone to error (which hopefully are corrected in follow-up studies). Even though, Boolean difference-making as spelled out in (BD) is not sufficient for causation, it corresponds to the most important Boolean constraint causes have to comply with. Factors that do not satisfy (BD) can be excluded as causes; correspondingly, Boolean solution formulas that contain (BD)-violating factors are not amenable to a causal interpretation. By contrast, a causal interpretation of solution formulas that are exclusively composed of factors complying with (BD) and that are inferred from competently collected data is (inductively) warranted. I shall say that such solution formulas are *causally interpretable*.

I do not intend to claim here that all causal structures empirically manifest themselves in terms of Boolean difference-making. In fact, according to so-called *causal pluralism* (cf. e.g. Psillos 2009), difference-making and power theories of causation do not mutually exclude each other, rather, they simply highlight different aspects of causation. What I do intend to claim, though, is that Boolean difference-making is the feature of causation that is tracked by Boolean methods of causal inference as QCA. Hence, whoever applies a Boolean method, searches for causal dependencies that comply with (BD). To uncover causal structures that do not exhibit relations of Boolean difference-making, recourse to different methods must be made.

To sum up, this section has revealed that only redundancy-free sufficient and necessary conditions feature Boolean difference-makers and, thus, comply with the core Boolean criterion for causation. In consequence, a Boolean method of *causal* data analysis must seek for minimally sufficient and minimally necessary conditions of scrutinized outcomes.

3 Inverse Search

This section aims to show that in order for Boolean methods to output solution formulas that are causally interpretable parsimony must be maximized. Only maximally parsimonious solution formulas identify minimally sufficient and minimally necessary conditions and, as only the latter can correspond to causal conditions, only maximally parsimonious solution formulas can mirror causal structures.

To this end, I will perform so-called *inverse searches*, which—as the label suggests—reverse the order of causal discovery as it is normally conducted in scientific practice. An inverse search involves three steps: (i) a causal structure σ is presumed to be given, (ii) artificial data δ is generated by letting the factors in σ behave/operate in accordance with σ, (iii) δ is processed by a method of causal data analysis with the aim of recovering σ. If, and only if, the scrutinized method finds σ, the inverse search is successfully completed. Even though successful inverse searches are standardly implemented as quality benchmarks for methods of causal discovery (cf. e.g. Spirtes, Glymour, and Scheines 2000; Pearl 2000), they are hardly ever conducted in the literature on Boolean data analysis.

In real-life contexts of causal discovery there are manifold reasons as to why methods of data analysis fail to find the causal structure that actually underlies processed data. A researcher may fail to properly control for background influences, to the effect that collected data is too noisy, that is, confounded by unmeasured causes of an investigated outcome. Or if a process is being analyzed whose relevant factors—for whatever reasons—cannot be manipulated or influenced artificially, the number or diversity of data points that nature happens to provide may be wanting. Or the selection, calibration, or measurement of analyzed variables may be flawed, and so forth. Yet, all of these reasons for failing to find causal structures are due to problems of *data collection* and *pre-processing*, and thus to a task that must be completed prior to the actual analysis of the data. No method can be expected to successfully uncover causal structures based on fragmentary, confounded, or otherwise flawed data. Outputs of procedures of causal data analysis are only as good as the analyzed data.

Therefore, in order to evaluate causal inference procedures and corresponding search strategies, problems of data collection/pre-processing must be factored out by assumption (or idealization). The tool of an inverse search lends itself particularly well to this purpose. Based on the causal structure that is presupposed as given in an inverse search, we can artificially generate data that is free of confounding noise, diversity limitations, selection bias or calibration and measurement errors, that is, we can produce *ideal data*. Against such an idealized background, a failure to find the structure based on which the data was generated can then be directly and unambiguously ascribed to a deficiency in the implemented method or search strategy.

The previous section has already laid the basis for an inverse search that avoids problems of data collection. Figure 1 provides a causal structure with a list of all and only the configurations of the factors involved in that structure that can possibly be observed relative to a homogenous (or fixed) setting of unmeasured background causes. Also, there are no rows featuring one and the same configuration of conditions combined with both the presence and the absence of outcome E.[6] We are hence justified in presuming a proper variable calibration and the absence of data confounding and measurement error. Moreover, we can assume that for all configurations in table 1(b) we have a sufficient amount of cases in our raw data to meet inclusion cut-offs. All in all, we shall assume that table 1(b) represents an *ideal truth-table* for a Boolean analysis—to the effect that an adequate Boolean method of data analysis must be able to find the causal dependencies in structure 1(a) based on table 1(b). Or differently, if a method or search strategy does not find the causes of E based on table 1(b), it is inadequate *qua* procedure of causal inference.

For the purposes of an inverse search the interpretation of the factors involved in structure 1(a) does not matter. All that matters is that the behavior of the factors in the set $\mathbf{V} = \{A, B, C, D, E\}$ is assumed to be regulated by structure 1(a). Nonetheless, in order to substantiate that a structure as 1(a) is very commonplace in the field of social sciences, the following would be a viable interpretation of the factors in $\mathbf{V}$:

A:	high share of native population	D:	high GNP	
B:	strong unions	E:	high level of education	$(\mathcal{I})$
C:	strong left parties			

If $(\mathcal{I})$ constitutes our research context, the relevant question we are going to want to answer in the following is whether a high level of education (E), say, in western democracies, is caused only be a high share of native population (A) and, alternatively, a high GNP (D) or whether strong unions (B) and strong left parties (C) causally contribute as well. The correct answer, according to the assumed structure in figure 1(a) is the former option, that is, A and D are the only two alternative causes of E.

As anticipated in the introduction, the standard methodological framework for Boolean causal data analysis, QCA, provides different search strategies whose main differences concern the way in which redundant elements are removed from solution formulas. More specifically, Ragin and Sonnett (2005) distinguish between a conservative strategy $\mathcal{S}_1$, an intermediate strategy $\mathcal{S}_2$, and a liberal strategy $\mathcal{S}_3$. We subsequently apply these three search strategies to table 1(b) with the aim of recovering the causes of E given in structure 1(a). It will turn out that only $\mathcal{S}_3$ successfully completes this inverse search.

[6] In the QCA literature, two rows c_i and c_h in a truth-table such that one and the same configuration of conditions is combined with the presence of the outcome in c_i and with its absence in c_h are often very misleadingly called "contradictory" (cf. e.g. Rihoux and De Meur 2009; Schneider and Wagemann 2012, §5.1; Rubinson 2013). In fact, however, such a pair of rows is far from being contradictory, rather, it merely entails that the relevant configuration of conditions is neither sufficient for the presence nor for the absence of the outcome.

As structure 1(a) and table 1(b) involve binary variables only, we can confine our analysis to crisp-set QCA (csQCA). I assume that the reader is familiar with the procedural details of csQCA (cf. Ragin 1987; Rihoux and De Meur 2009). Still, to clearly understand why a search strategy does or does not complete our inverse search, it is required that we briefly review the computational core of QCA that is concerned with removing redundancies from sufficient and necessary conditions, i.e. the procedural part of QCA whose function it is to turn mere sufficient and necessary conditions into *causally interpretable* sufficient and necessary conditions. Both in csQCA and fuzzy-set QCA (fsQCA) this core is constituted by Quine-McCluskey optimization (Q-M), which is a procedure to minimize Boolean expressions standardly used in electrical engineering or digital logic design (cf. Quine 1959). The operational details of Q-M are best presented by means of concrete examples.

Let us hence eliminate redundancies from an exemplary sufficient condition of E in table 1(b) by virtue of Q-M. The configuration $ABCD$, which is combined with E in row c_1, is sufficient for E, because table 1(b) does not contain a row where $ABCD$ is combined with $\overline{E}$. To determine whether $ABCD$ contains redundancies, Q-M parses the input table 1(b) to find other rows that accord with c_1 in regard to the outcome and all other factors except for one. Such a row with exactly one difference is easily found. In c_2, E is combined with the configuration $ABC\overline{D}$, which accords with $ABCD$ in all factors except for D. The pair of rows $\langle c_1, c_2 \rangle$ reveals that, in the context of ABC, E occurs both if D is given and if it is not. In that context, D makes no difference to E and is hence redundant. Therefore, Q-M eliminates D from $ABCD$ and $\overline{D}$ from $ABC\overline{D}$ to yield ABC. Similarly, the configuration in row c_3 coincides with the one in row c_4 in all factors except for D which is present in c_3 and absent in c_4. Consequently, Q-M removes D and $\overline{D}$ from the corresponding sufficient conditions of E to yield $A\overline{B}C$. Next, since a comparison of the two sufficient conditions, ABC and $A\overline{B}C$, that result from the two previous minimization steps reveals that B makes no difference to E in contexts that feature AC, Q-M continues to eliminate B and $\overline{B}$, respectively—and so forth, until no further redundancies are found.

The feature of this minimization procedure that is of crucial importance for our purposes is that Q-M only eliminates conjuncts of a sufficient condition if the corresponding truth-table actually contains a pair of rows that accord with respect to the outcome as well all factors *except for one*. If such a pair of rows does not exist for a particular sufficient condition, the latter cannot be further minimized. To facilitate later reference to this restriction, I label it the *one-difference restriction*.

In light of the one-difference restriction, eliminating *all* redundancies from sufficient conditions by means of Q-M presupposes that the analyzed truth-table exhibits high diversity with respect to the logically possible configurations of potential cause factors (conditions). Consider, for example, row c_7 of table 1(b) which features E in combination with the configuration $\overline{ABC}D$. As there is no row where $\overline{ABC}D$ is combined with $\overline{E}$, $\overline{ABC}D$ is sufficient for E. However, table 1(b) does not contain a row that accords with c_7 with respect to the outcome and all conditions except for one. In consequence, although it can easily be seen (even without algorithmic help) that all rows of table 1(b) that feature D also feature E, i.e. that D itself is sufficient for E, Q-M cannot eliminate the redundant $\overline{ABC}$ from $\overline{ABC}D$ based on the configurations contained in table 1(b).

A table as 1(b) that does not comprise all 2^n logically possible configurations of n conditions of an investigated outcome is called *limitedly diverse* in the QCA terminology (Ragin 2000, 139). Logically possible configurations that are missing from analyzed truth-tables are termed *logical remainders*. In cases of limited diversity, QCA offers the researcher the possibility to counterfactually add logical remainders based on her available theoretical knowledge about an investigated process. The three aforementioned QCA search strategies essentially differ with respect to how much restrictions they impose on the counterfactual introduction of remainders. The conservative strategy $\mathcal{S}_1$ prohibits the introduction of counterfactual configurations altogether, the intermediate strategy $\mathcal{S}_2$ allows for the introduction of so-called *easy* (Ragin 2008a) or *tenable* (Schneider and Wagemann 2012) counterfactuals, and according to the liberal strategy $\mathcal{S}_3$ any counterfactuals, easy/tenable and difficult/untenable ones, may be introduced that contribute to maximizing parsimony of resulting solution formulas. Counterfactuals that are introduced for the purpose of increasing parsimony are also called *simplifying assumptions*.

$\mathcal{S}_1$ and $\mathcal{S}_3$ are precisely defined search strategies whose algorithmic implementation is straightforward. The concrete blueprint for $\mathcal{S}_2$, by contrast, depends on what counterfactual configurations are deemed easy/tenable in a given research context. The easy/tenable counterfactuals are those remainders that are compatible with the available theoretical knowledge, the difficult/untenable counterfactuals are those that are not compatible therewith. That is, the distinction between easy/tenable and difficult/untenable counterfactuals is not stable but varies with the epistemic context. Moreover, there are various variants of $\mathcal{S}_2$ available in the literature (Schneider and Wagemann 2012, chaps. 6, 8): (i) $\mathcal{S}_2$ of the so-called *Standard Analysis* (SA), (ii) $\mathcal{S}_2$ of the *Enhanced Standard Analysis* (ESA), (iii) $\mathcal{S}_2$ of the *Theory-Guided Enhanced Standard Analysis* (TESA). Their main differences concern the complexity of added counterfactuals, the manner in which they are brought to bear—as so-called *directional expectations* or as truth-table rows—, and at what point within a complete QCA of a truth-table these simplifying assumptions are introduced. These variations will not be relevant for our purposes, because it will turn out that variants of $\mathcal{S}_2$ only pass our inverse search test when they are equivalent to $\mathcal{S}_3$.

To see this, let us begin by applying $\mathcal{S}_1$ to table 1(b). When processed in terms of $\mathcal{S}_1$, QCA issues this solution formula:[7]

$$AC \vee BCD \vee \overline{ABC}D \leftrightarrow E \tag{4}$$

As 1(b) is an ideal and noise-free truth-table, (4) is a perfect Boolean solution formula, that is, it has a maximal consistency and coverage of 1 (cf. Ragin 2006), which I express by means of the biconditional operator in (4).

(4) illustrates what we anticipated already: if no remainders are counterfactually added to table 1(b), Q-M does not succeed in eliminating all redundancies. In particular, no redundancies at all can be removed from $\overline{ABC}D$. Due to the fact that not all redundancies are eliminated, (4) does not identify Boolean difference-makers of E. Although there do not exist any circumstances in which B and C make a difference to E, they appear as parts of sufficient conditions of E and they are contained in a (disjunctive) necessary condition of E. In consequence, (4) does not adequately represent the causes of E in structure 1(a). A causal interpretation of (4) would entail that both C and B causally contribute to E, where in fact they do not. Expressed in terms of research context ($\mathcal{I}$), even though strong left parties and unions are not causes of a high level of education in the presupposed structure 1(a), a causal interpretation of (4) would erroneously entail that the former cause the latter after all. In sum, the solution formula of $\mathcal{S}_1$ must not be causally interpreted.

By contrast, $\mathcal{S}_3$ succeeds in finding the true causes of E in structure 1(a) based on table 1(b). The solution formula for outcome E produced by $\mathcal{S}_3$ is this:

$$A \vee D \leftrightarrow E \tag{5}$$

[7] The QCA solution formulas in this article were built using the R-implementation of QCA by Alrik Thiem and Adrian Duşa, version 1.0-5 (Thiem and Duşa 2013a; 2013b).

case	A	B	C	D	E
c_9^*	0	0	1	1	1
c_{10}^*	0	1	0	1	1
c_{11}^*	1	0	0	0	1
c_{12}^*	1	0	0	1	1
c_{13}^*	1	1	0	0	1
c_{14}^*	1	1	0	1	1
$[c_{15}]$	0	0	0	0	1
$[c_{16}]$	0	0	1	0	1

Table 2 List of logical remainders for outcome E in table 1(b). Configurations c_9^* to c_{14}^* (where "*" marks counterfactuality) are required by $\mathcal{S}_3$ in order to eliminate all redundancies from sufficient and necessary conditions of E. Configurations $[c_{15}]$ and $[c_{16}]$ are irrelevant.

By counterfactually adding any remainders that contribute to maximizing parsimony, Q-M manages to eliminate all redundancies from sufficient and necessary conditions of E and, thus, to identify Boolean difference-makers as defined in (BD). Moreover, as 1(b) contains optimal data, a causal interpretation of (5) is warranted; it reveals exactly the two alternative causes, A and D, that E actually has in the underlying structure 1(a). Against the background of research context ($\mathcal{I}$), $\mathcal{S}_3$ correctly determines that a high share of native population and a high GNP are the two alternative causes of a high level of education. $\mathcal{S}_3$ hence successfully completes our inverse search.

To this end, $\mathcal{S}_3$ counterfactually introduces the remainders given in rows c_9^* to c_{14}^* of table 2. Those are the configurations that Q-M needs in order to find the correct causal model (5). That, in turn, means that variants of the intermediate strategy $\mathcal{S}_2$ only find solution formula (5) if they likewise counterfactually add all configurations c_9^* to c_{14}^*. If any of those configurations is deemed difficult/untenable and, thus, not introduced by $\mathcal{S}_2$, the latter outputs a solution formula with at least some redundancies, i.e. with at least some factors that are not Boolean difference-makers and, hence, not causes. As indicated above, whether a configuration is difficult/untenable or not depends on the theoretical background knowledge that is available in a given research context—and available background knowledge may vary from context to context.

However, note that we started our inverse search with ideal and, in particular, *complete* data. Table 1(b) contains all empirically possible configurations of the factors in the set $\mathbf{V} = \{A, B, C, D, E\}$, given that their behavior is regulated by the structure in figure 1(a). In other words, most remainders in table 2 are deemed empirically *impossible* by structure 1(a). To illustrate, consider configuration c_{10}^*, where B is given but C is not. This configuration is not compatible with structure 1(a) because, according to the latter, B is a sufficient cause of C, meaning that whenever B is given, so is C. Or in c_{11}^* to c_{14}^* A is given without C, even though structure 1(a) entails that A is sufficient for C. According to the standard analysis of the notion of knowledge in terms of *justified true belief* (cf. Ichikawa and Steup 2013), nothing that is impossible can possibly be known. Hence, a researcher analyzing structure 1(a) cannot have theoretical knowledge about that structure that would identify configurations c_{10}^* to c_{14}^* as easy/tenable counterfactuals.

The only remainder in table 2 that might still pass as an easy/tenable counterfactual and that contributes to eliminating redundancies is c_9^*. The rationale for such an assessment is as follows. The data in table 1(b) is assumed to be collected against a homogenous background of unmeasured causes of the effects in structure 1(a). That means, in particular, that causes of C that are not contained in the set $\mathbf{V}$ are assumed to be absent in the background against which the data in table 1(b) is collected. But that does not exclude that there might be different background configurations of unmeasured causes such that C is present without any of its causes in $\mathbf{V}$ being present. Hence, it might be that A and B are not the only causes of C. In consequence, configurations featuring C in combination with $\overline{AB}$ might be considered empirically possible against a different causal background than the one behind table 1(b), such that remainder c_9^* could be deemed easy/tenable. If $\mathcal{S}_2$ counterfactually introduces c_9^*, it outputs this solution formula:

$$AC \vee CD \vee \overline{AB}D \leftrightarrow E \tag{6}$$

As can easily be seen, (6) fares a bit better than (4), *viz.* the output of $\mathcal{S}_1$. Nonetheless, the sufficient and necessary conditions in (6) still feature numerous elements that are not Boolean difference-makers and, thus, not causes of E. A causal interpretation of (6) would erroneously entail that C and $\overline{B}$ causally contribute to E. Overall, a variant of $\mathcal{S}_2$ that counterfactually adds only a proper subset of the remainders c_9^* to c_{14}^* to table 1(b) fails to find structure 1(a) and, hence, does not pass our inverse search test.

As the distinction between easy/tenable and difficult/untenable counterfactuals cannot be conclusively fixed, friends of a variant of $\mathcal{S}_2$ might insist that all relevant remainders c_9^* to c_{14}^* in table 2 could be deemed easy/tenable, for the causal structure regulating the behavior of the factors in $\mathbf{V}$ could have been different. They might hold that causal laws governing the world we live in do not hold of necessity. Of course, if the causal dependencies among the factors in $\mathbf{V}$ had been different, the latter could have been configured in any logically possible manner. Whether causal laws hold of necessity or not is a deep metaphysical question on which I do not want to dwell here. All that matters for our current purposes is that, if any logical remainder that $\mathcal{S}_3$ needs to maximize parsimony is considered difficult/untenable, the solution formulas output by variants of $\mathcal{S}_2$ fail to identify Boolean difference-makers, that is, causes. The output of $\mathcal{S}_2$ is causally interpretable only if it is identical to the output of $\mathcal{S}_3$. Or differently, $\mathcal{S}_2$ is a strategy for *causal* discovery only if it is equivalent to $\mathcal{S}_3$, which—as we have seen above—dispenses with the problematic distinction between easy/tenable and difficult/untenable counterfactuals altogether.

The only QCA search strategy that, if applied to non-flawed data, is guaranteed to output solution formulas that represent causal structures is the liberal strategy $\mathcal{S}_3$. In light of section 2, this result is not surprising. Section 2 has shown that parsimony is not simply a pragmatic or aesthetic virtue of QCA solution formulas, which can be dispensed with if it comes at a high price. Rather, there is a tight conceptual interdependence between parsimony and causality. According to the notion of causation presupposed by Boolean search methods, causes are Boolean difference-makers of their effects. Only factors that are contained in minimally sufficient and minimally necessary conditions are Boolean difference-makers, and only maximally parsimonious solution formulas feature minimally sufficient and minimally necessary conditions. That means only maximally parsimonious solution formulas are causally interpretable. If QCA is applied in order to discover causal dependencies or to test causal hypotheses, neither the conservative search strategy $\mathcal{S}_1$ nor any of the available variants of the intermediate strategy $\mathcal{S}_2$ must be implemented; rather, recourse to $\mathcal{S}_3$ is called for.

4 Parsimony vs. Tenability

In light of the fact that causal data analysis, in general, and the testing of causal hypotheses, in particular, are among the main purposes for which QCA is currently being applied in social scientific practice the question arises as to why the most liberal strategy $\mathcal{S}_3$, which was the principal search strategy Ragin suggested in (1987), was ever supplemented by further strategies. In fact, in recent years it has become more and more common that intermediate solution formulas are presented as the preferable QCA outputs (cf. e.g. Ragin 2008b, 171-172).[8]

The main reason for the widespread turning away from maximally parsimonious solutions, as e.g. Schneider and Wagemann (2012, chs. 6, 8) convincingly show, is that QCA's liberal strategy $\mathcal{S}_3$ faces a serious problem. In order to eliminate all redundancies, $\mathcal{S}_3$ is regularly forced to introduce *untenable* simplifying assumptions, i.e. configurations that are empirically impossible because they contradict causal or logical laws (or simply our general world knowledge). We have already seen an instance of this problem in the previous section: even though A is a sufficient cause of C in structure 1(a), Q-M—due to the built-in one-difference restriction—can only reveal that C is not a Boolean difference-maker of E in table 1(b) if the configuration $A\overline{C}$ is counterfactually introduced, which however is determined to be impossible by the very causal structure under investigation.[9] Or Schneider and Wagemann's (2012, ch. 8) "pregnant man" is another telling example: in a study that investigates how gender, taking the pill, and being pregnant causally contribute to, say, suffering from thrombosis, $\mathcal{S}_3$ will at some point be forced to counterfactually introduce the combination of being a man and being pregnant, which of course is a biological impossibility.

Clearly, a method for causal data analysis that infers a causal structure based on counterfactual data points that contradict the very structure under investigation or that is forced to assume what is known to be impossible has more than an air of fishiness. Such a method runs the risk of complete trivialization. From assumptions that cannot possibly be true, i.e. that are necessarily false, everything can be inferred (*ex falso quodlibet*). More concretely, from the assumption that a man is pregnant it can both be inferred that taking the pill is a sufficient cause of thrombosis and that taking the pill is *not* a sufficient cause of thrombosis. A necessarily false assumption entails any arbitrary causal model (as well as the negation thereof). That is, on pain of trivialization, no search strategy for causal data analysis must be allowed to introduce necessarily false simplifying assumptions. Yet, to repeat it once again, in order to eliminate all redundancies from solution formulas, Q-M regularly forces $\mathcal{S}_3$ to do just that.

Many studies concerned with developing or applying QCA acknowledge a *tension* between parsimony and tenability of simplifying assumptions. That, however, is a gross understatement. When applied as a method for causal data analysis, the polarity between parsimony and tenability presents QCA with a pressing *dilemma*: the causal interpretability of solution formulas calls for maximal parsimony, but in order to reach the latter by means of Q-M, necessarily false assumptions may be required, which, in turn, trivializes corresponding causal inferences. In view of the fact that the tight conceptual connection between parsimony, Boolean difference-making, and causation has gone unnoticed so far in the QCA literature, most authors prioritize tenability of simplifying assumptions over parsimony without noticing that this maneuver prohibits the causal interpretability of resulting solution formulas.

[8] Without providing any reasons, Rubinson (2013, 2866) rightly deplores the fact that "far too many researchers automatically pick the intermediate solution, assuming that it must be best". The most straightforward reason why a general preference of intermediate solutions is deplorable is simply that these solutions are not (causally) explanatory.

[9] A causal dependence among investigated conditions is but one reason why certain remainders turn out to be impossible—logical, conceptual, or mereological dependencies being further reasons. Thus, the arrow from A to C in figure 1(a) could also be interpreted in terms of such a non-causal form of dependence.

However, prioritizing tenability over parsimony and thereby biting the bullet that QCA can no longer be used as a method of causal data analysis is *not* the only way out of the above dilemma. Maximal parsimony and tenability of the assumptive basis of Boolean data analysis can both be had at the same time. What is responsible for the QCA dilemma between parsimony and tenability is not some in-principle incompatibility of parsimony and tenability, rather the culprit is QCA's reliance on Quine-McCluskey optimization (Q-M), which imposes the one-difference restriction on data diversity.

Q-M is a procedure that was originally developed for the sole purpose of simplifying Boolean or truth-functional expressions (formulas). What is crucial for Q-M is that the expression that is input into the procedure and the expression that is output by the procedure are *logically equivalent*, i.e. that input and output have exactly the same meaning (cf. Quine 1959).[10] To guarantee for logical equivalence, Q-M is very cautious in eliminating elements from input expressions—thus the one-difference restriction. Yet, inputs and outputs of a Boolean procedure of causal inference do not have to be logically equivalent. More concretely, Boolean solution formulas do not have to be logically equivalent with the processed truth-table. Rather, the former must identify the causes that are operative in the data behind the latter. As a causal inference procedure does not strive for logical equivalence of input and output but merely for redundancy-freeness of the output, there is no need at all to draw on Q-M when it comes to eliminating redundancies. On the contrary, since QCA's reliance on Q-M is what is responsible for the dilemma between parsimony and tenability, the proper way around that dilemma is to dismiss Q-M when it comes to identifying Boolean difference-makers. Solution formulas of Boolean methods must be freed of redundancies by procedures that do not impose the one-difference restriction.

And, as a matter of fact, there exist alternatives to Q-M. Baumgartner (2009a; 2009b) proposes so-called *Coincidence Analysis* (CNA) as an alternative Boolean method of causal data analysis. CNA shares all of QCA's basic goals and intentions: it focuses on configurational complexity rather than on net effects (which are scrutinized by standard quantitative methods), it processes the same kind of data as QCA, i.e. small- to intermediate-N configurational data, and it also searches for causal dependencies defined in terms of Boolean difference-making.

There are two main differences between CNA and QCA. First, while QCA is designed to treat exactly one factor Z_i as outcome and all other factors in an analyzed truth-table as potential direct causes of Z_i, CNA can treat any number of factors in a set $\{Z_1, \ldots, Z_i\}$ as outcomes. That is, CNA does not only search for direct causal dependencies among $Z_1, \ldots, Z_{i-1}$, on the one hand, and Z_i, on the other, but also for dependencies among the conditions $Z_1, \ldots, Z_{i-1}$ themselves. Second, CNA does not remove redundant factors on the basis of Q-M but implements its own minimization algorithm that is custom-built for the discovery of complex causal structures.

This is not the place to reiterate the procedural details of CNA (cf. Baumgartner 2009a; 2009b; Baumgartner and Epple forthcoming). Let me just briefly indicate the basic ideas behind CNA's minimization algorithm. If there exist (deterministic) causal dependencies among n factors, it follows that not all 2^n logically possible configurations of these factors are also empirically possible. Causal dependencies constrain the range of empirical possibilities. To do justice to this trademark feature of causality, CNA infers causal dependencies

[10] In fact, the disjunction of solution terms of the conservative formula (4) is logically equivalent to the disjunction of configurations in rows c_1 to c_5 and c_7 in table 1(b), which are the configurations that are sufficient for E in that table. Neither the intermediate nor the parsimonious solution—(6) and (5)—preserve this logical equivalence.

not only from the configurations actually contained in truth-tables (as does QCA) but also from the fact that certain configurations *are not contained therein*.

To determine whether, say, a complex sufficient condition ABC of a factor E contains redundancies or is minimally sufficient, CNA systematically eliminates conjuncts from ABC. For each conjunction that results from such an elimination, say for BC, CNA then parses a corresponding truth-table $\mathcal{T}$ to check whether $\mathcal{T}$ contains BC in combination with the absence of E, i.e. $\overline{E}$. If $\mathcal{T}$ does not contain the configuration $BC\overline{E}$, BC is itself sufficient for E, which means that A is redundant. CNA then proceeds to eliminate the next conjunct from BC and tests for further redundancies, until no more redundancies are found. By contrast, if $\mathcal{T}$ contains the configuration $BC\overline{E}$, BC is not itself sufficient for E, which means that A makes a difference to E and is, thus, not redundant. Accordingly, CNA re-adds A to BC and proceeds to eliminate B, and so forth. That is, while Q-M only eliminates factors from sufficient conditions if the latter reduced by a respective factor is actually contained in the truth-table in a way that satisfies the one-difference restriction, CNA eliminates factors from sufficient conditions if the latter reduced by a respective factor is not contained in the truth-table in combination with the absence of a corresponding outcome.[11]

Similarly, to determine whether a complex necessary condition $A \vee B \vee C$ of an outcome E is minimally necessary, CNA systematically eliminates disjuncts from $A \vee B \vee C$ and checks for every resulting disjunction, say for $B \vee C$, whether it is still necessary for E, i.e. whether $\mathcal{T}$ contains a configuration featuring E without any of the disjuncts in $B \vee C$. If the truth-table does not contain such a configuration, $B \vee C$ is still necessary for E, which means that the eliminated disjunct A is redundant. Next, $B \vee C$ is tested for further redundancies, until no more redundancies are found.

As CNA does not impose the one-difference restriction, limited data diversity in no way hampers CNA's capacity to remove all redundancies from sufficient and necessary conditions. By taking into account the configurations not contained in a truth-table, CNA can systematically test for redundancies without ever being forced to counterfactually introduce missing configurations.[12] In sum, due to its reliance on Q-M, QCA is forced to eliminate redundancies from solution formulas on the basis of problematic counterfactual assumptions as "Had the configuration $AB\overline{C}$ occurred, the outcome E would have occurred as well". CNA, in contrast, eliminates redundancies from solution formulas based on unproblematic negative existential claims about an analyzed truth-table $\mathcal{T}$, for example, "The configuration $A\overline{B}\overline{C}\overline{E}$ is not contained in $\mathcal{T}$".[13]

In this manner, CNA always outputs maximally parsimonious solution formulas that identify Boolean difference-makers, which—when inferred from competently collected data—are causally interpretable. Moreover, as CNA does not only search for the dependen-

[11] Eliason and Stryker (2009, 126) implement a very analogous minimization idea in the context of a goodness-of-fit test for fuzzy-set solutions.

[12] Note that CNA does not preclude the addition of counterfactual configurations; it just does not require it. If there are good theoretical grounds for counterfactually supplementing the data, CNA does not prevent the researcher from doing so. Whereas in the case of QCA it is the algorithmic machinery of the method that calls for the introduction of counterfactual configurations in order to eliminate all redundancies from solution formulas, CNA leaves counterfactual considerations entirely up to the researcher's background theory.

[13] The fact that QCA's $\mathcal{S}_3$ and CNA infer the same causal model for outcome E from table 1(b) does not indicate that corresponding inferences are based on the same or even related assumptions. One and the same conclusion can be inferred from very different assumptions. For instance, "Socrates is mortal" can be inferred from the assumptions "Socrates is a man" and "All men are mortal", or from "All immortal things are angels" and "Socrates is not an angel", or from any contradiction, e.g. from "It rains and it does not rain". A conclusion is established if it is not only validly inferred from a set of assumptions, but if the latter are moreover cogently justifiable.

cies among one designated outcome and the rest of the factors in an analyzed truth-table, but for all dependencies of minimal sufficiency and necessity among all the involved factors, it not only correctly uncovers the causes of E in structure 1(a) but also the causes of C. When given table 1(b) as input, CNA outputs this complex solution formula (where the concatenation of the atomic solutions for C and E indicates conjunction):

$$(A \vee B \leftrightarrow C)(A \vee D \leftrightarrow E) \tag{7}$$

This result substantiates that QCA's dilemma between parsimony and tenability of simplifying assumptions does not have to be resolved by dispensing with maximal parsimony and, thereby, endangering the causal interpretability of inferred solution formulas. By replacing Q-M by its own custom-built minimization algorithm, CNA succeeds in maximizing parsimony on the basis of completely unproblematic and, hence, easily tenable assumptions.

5 Conclusion

The first part of this paper has shown that there is a tight conceptual connection between parsimony and causality. Parsimony is not simply a pragmatic virtue that facilitates the comprehensibility or readability of Boolean causal models and that can be dispensed with if it comes at a high price. Rather, parsimony is essential for the causal interpretability of models output by Boolean methods. A factor A is a cause of another factor E only if there exist circumstances in which the presence and absence of A makes a difference to the presence and absence of E. Only solution formulas that exclusively feature Boolean difference-makers as defined in (BD) are amenable to a causal interpretation.

In the second part, I then conducted a simple inverse search revealing that only the most liberal search strategy $\mathcal{S}_3$ of QCA succeeds in correctly identifying Boolean difference-makers. If inferred from appropriate data, only the most parsimonious solution formulas of QCA are guaranteed to reflect causation. That is, if QCA is applied, as it often is, to generate causal explanations, to uncover causal structures or to test causal hypotheses, recourse must be made to the liberal search strategy $\mathcal{S}_3$. However, as is well recognized in the literature, $\mathcal{S}_3$ runs the risk of trivializing causal inferences by introducing untenable, *viz.* necessarily false, simplifying assumptions.

Finally, we saw that it is Quine-McCluskey optimization (Q-M) that presents QCA with the dilemma between parsimony and tenability of simplifying assumptions. A Boolean method of causal data analysis, as CNA, that does not eliminate redundancies from solution formulas by means of Q-M manages to maximize parsimony and, thereby, to correctly uncover causal structures without drawing on untenable counterfactual assumptions.

Let me end by reemphasizing a caveat. None of the findings of this paper shall be taken to imply that parsimonious Boolean solutions *always* correctly mirror underlying causal structures. If the data processed by any Boolean method is deficient, parsimonious solutions will tend to miss the target just as any other type of solutions. Yet, while conservative and intermediate solutions are even off-target when they have been generated from ideal data, to the effect that the corresponding search strategies of QCA under no circumstances output causally interpretable solutions, there is a positive chance that parsimonious solutions truthfully reflect underlying causal structures, *viz.* whenever the analyzed data is of the required quality. All in all, thus, studies that aim for causally explanatory solutions *must*, under all circumstances, present the parsimonious solutions as their main results—and not, as is currently customary in the QCA literature—intermediate solutions. Moreover, if parsimony can only be maximized at the prize of introducing untenable simplifying assumptions, recourse must be made to a Boolean method that dispenses with Quine-McCluskey optimization.

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